The constraints of a straightforward soundness theorem (Theorem 1) lead to an abstract transition relation over this space:

$$
\begin{aligned}
((\llbracket (f\ e_1 \ldots e_n)^{\ell} \rrbracket, \hat{\beta}, \widehat{ve}, \hat{t}), \equiv) \rightsquigarrow ((call, \hat{\beta}'', \widehat{ve}', \hat{t}'), \equiv'), \text{ where:}&\\
\hat{d}_i &= \hat{\mathcal{E}}(e_i, \hat{\beta}, \widehat{ve})\\
\hat{d}_0 &\ni (\llbracket (\lambda^{\ell'}\ (v_1 \ldots v_n)\ call) \rrbracket, \hat{\beta}')\\
\hat{t}' &= \widehat{tick}(call, \hat{t})\\
\hat{b}_i &= \widehat{alloc}(v_i, \hat{t}')\\
\hat{B} &= \left\{ \hat{b}_i : \hat{b}_i \in \widehat{Bind}_1 \right\}\\
\hat{\beta}'' &= (\hat{g}_{\hat{B}}^{-1} \hat{\beta}')[v_i \mapsto \hat{b}_i]\\
\widehat{ve}' &= (\hat{g}_{\hat{B}}^{-1} \widehat{ve}) \sqcup [\hat{b}_i \mapsto (\hat{g}_{\hat{B}}^{-1} \hat{d}_i)],
\end{aligned}
$$

and singleton bindings are reflexively equivalent:

$$
\frac{\hat{b} \in \widehat{Bind}_1}{\hat{b} \equiv' \hat{b},}
$$

and bindings between singletons are trivially equivalent:

$$
\frac{\hat{\beta}(e_i) \in \widehat{Bind}_1 \qquad \hat{b}_i \in \widehat{Bind}_1}{\hat{\beta}(e_i) \equiv' \hat{b}_i,}
$$

and untouched bindings retain their equivalence:

$$
\frac{\hat{b} \equiv \hat{b}' \qquad \hat{b} \notin \hat{B} \qquad \hat{b}' \notin \hat{B}}{\hat{b} \equiv' \hat{b}',}
$$

and bindings re-bound to themselves also retain their equivalence:

$$
\frac{\hat{\beta}(e_i) \equiv \hat{b}_i}{\hat{\beta}(e_i) \equiv' \hat{b}_i.}
$$

4.1 Solving the generalized environment problem

Under the direct product abstraction, the generalized environment theorem, which rules on the equality of individual bindings, follows naturally:

Theorem 3. *Given a compound abstract state* $((call, \hat{\beta}, \widehat{ve}, \hat{t}), \equiv)$ *and two abstract bindings,* $\hat{b}$ *and* $\hat{b}'$*, if* $\dot{\alpha}^{\eta}(call, \beta, ve, t) \sqsubseteq ((call, \hat{\beta}, \widehat{ve}, \hat{t}), \equiv)$ *and* $\eta(b) = \hat{b}$ *and* $\eta(b') = \hat{b}'$ *and* $\hat{b} \equiv \hat{b}'$*, then* $ve(b) = ve(b')$*.*

Proof. By the structure of the direct product abstraction $\dot{\alpha}^{\eta}$.